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A Comparative Analysis of Means and Variances in Complex Urban Systems

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Abstract
The regional economics and geography literature has in recent years shown interesting conceptual and methodological contributions on the validity of Gibrat's Law and Zipf's Law. Despite distinct modeling features, they express similar fundamental characteristics in an equilibrium situation. Zipf's law is formalized in a static form, while its associated dynamic process is articulated by Gibrat's Law. Thus, it seems that both Zipf's Law and Gibrat's Law can be conceived of as heterozygote twins. Unfortunately, empirical investigations on the close relationship 'Gibrat's Law vs Zipf's Law' are rather rare.

The present paper aims now to answer the following research question: can (a generalization of) Gibrat's Law allow us to infer Zipf's Law, and vice versa? In our conceptual and applied framework, particular attention will be paid to the role of the mean and the variance of city population as key indicators for assessing the (non-)validity of the generalized Gibrat's Law.

Our empirical experiments are based on a comparative analysis between the dynamics of the urban population of five countries with entirely mutually contrasting spatial-economic characteristics: Botswana, Germany, Hungary, Luxembourg and Malta. We arrive at the following results: If (i) the mean is independent of city size (first necessary condition of Gibrat's law); and (ii) the coefficient of the rank-size rule/Zipf's Law is different from one, the variance is dependent on city size.

This finding suggests an important research implication: in modeling urban growth, Gibrat's law holds only with respect to the condition on the mean, but not on the variance, thus allowing for heterogeneity in the growth of small and big cities. Furthermore, differences in population growth lead to differences in the hierarchy of a city system (with a rank-size coefficient different from one); this phenomenon creates a possibility of asymmetric shocks affecting the distribution of big vs. small cities.

Key-words: rank-size rule, Zipf's law, (generalised) Gibrat's Law, hierarchical structure, spatial interaction, city growth

1.Gibrat's Law vs Zipf's Law: Preliminary Considerations

1.1. Preface

Cities all over the world offer an amazing variety in terms of size and growth rates. Despite these differences, systems of cities do not exhibit a random pattern, but a strict regularity in terms of urban hierarchies and inter-urban connectivity. The genesis of such hierarchical perspectives on city size and urban systems can already be found in the seminal contributions of Christaller (1933) and Lösch (1940). The validity of these frameworks has extensively been tested in subsequent statistical experiments in many countries around the world. The conceptual foundation for the existence of central place hierarchies rests on various pillars: agglomeration advantages in cities (depending on city size), smart specialization of industries (depending on scale advantages in different size classes of cities), and transportation and logistics costs (depending on distance frictions between cities or between cities and their hinterlands). Urban hierarchies and inter-urban connectivity are therefore two sides of the same coin (see Paelinck and Nijkamp 1976).

Clearly, it ought to be added that the spatial range of interurban linkages has extended drastically over recent decades. Whereas a century ago, most cities were at best part of an interlinked regional or national system, nowadays cities are often part of a globally connected network. The underlying globalisation force field is not a random system either, but strictly governed by economic efficiency determinants, in which globally connected service networks and commodity chains play a critical role (see for fundamental contributions Neal (2012), Newman (2010)). There is no doubt that we live in a highly connected world. This is highlighted in a study by Reggiani and Schintler (2005), who assert: "*Our modern world is in a continuous state of flux. Modern transport systems and the emerging new style behaviours have created an unprecedented rise in mobility, at all spatial levels. The ever rising mobility patterns apply to all types of movement work, business, shopping and leisure, as well as to freight transport. Globalisation certainly plays a key role in this dynamic framework*" (p.1). Globalisation – which is defined as a broad area of increasing internationalisation of markets, changing consumption patterns and shifting of industrial activities all over the world – appears to form a common denominator for consumer/user (economic) activities with an immediate impact, namely, the emergence of a highly interconnected, interdependent and complex system of networks, or the rise of a complex network society. Consequently, mobility and migration – though continuously changing – lead to the increasing phenomenon of urban agglomerations: "*Agglomeration and residential mobility of the population between different geographic locations are tightly connected to economic activities*"

(Eeckhout, 2004, p. 1429). Surprisingly, despite the complex evolution of current socio-economic spatial networks, two robust empirical regularities seem to hold: Gibrat's law affirming that city growth does not depend on size, and Zipf's law stating the proportionality of a given city size to its rank.[1]

In the field of spatial economics, these two regularities have given rise, especially since the late '90s, to an increasing number of empirical studies, testing cities and economic growth at various spatial levels (national, regional, local), by means of Gibrat's law and Zipf's law. In the majority of urban studies, Zipf's law and Gibrat's law are generally confirmed by empirical data. For example, Eeckhout (2004) and Gonzalez-Val (2010) test the validity of both laws for all US cities: the former in the period 1990-2000, and the latter over the period 1900-2000; Ioannides and Overman (2003) and Gabaix and Ioannides (2004) examine Gibrat's and Zipf's laws in the US metropolitan areas over the same period (1900-1990); and Giesen and Suedekum (2011) investigate both these laws, by considering a sample of German cities with a population greater than 100,000 in the period 1975-1997. In contrast, only a few studies seem to reject these two empirical regularities, in particular, Black and Henderson (2003), who reject Gibrat's law over a sample of US metropolitan areas in the period 1900-1990, and Gonzalez-Val et al. (2012), who in their study of all cities in the US, Italy and Spain over the 20th century, find weak evidence of Gibrat's law. These contrasting results have prompted a continuous debate in the literature, on the (non)validity of Gibrat's law and/or Zipf's law. In this context, recent work has shown that Gibrat's law can be generalized, in the sense that only the mean of the city growth is independent from city size, while its variance can change according to size (Cordoba, 2003). Gibrat and Zipf have offered strong evidence on population dynamics and hierarchies, respectively, boiling down to 'simple' modeling perspectives on the evolution of urban systems in a given country. These two laws are often analyzed together, given their possible complementarity. Indeed, Champernowne (1953) and Simon (1955) have shown that rank-size distributions arise naturally, if Gibrat's law is satisfied. Gabaix (1999) has demonstrated that Gibrat's law leads to a Zipf distribution, while Cordoba (2003) argues that a weak version of Gibrat's law leads to more general rank-size distributions. Clearly, the question emerges whether Zipf's law (or its more general rank-size rule) is only a static version of the more dynamic Gibrat's law. Strictly speaking, it might be hypothesized that Zipf's law and the rank-size rule show features of monozygotic twins, while Zipf's law and Gibrat's law show features of heterozygotic twins, as the latter law is able to reveal distinct modeling outcomes, even though in equilibrium it

1 Another way to refer to Zipf's Law is a Pareto distribution, with a shape parameter equal to 1. It is investigated using the so-called rank-size rule. We note here that the slope coefficient of the rank-size rule represents the inverse form of the parameter of the conventional Pareto distribution. For more details, we refer inter alia to Adamic (2000) and Parr (1985). In this paper we refer to Zipf's law (Zipf's distribution), when the rank-size coefficient is exactly equal to 1. In all the other cases we refer to the rank-size rule (rank-size distribution).

expresses identical fundamental characteristics. A test of this proposition calls for evidence-based research. Starting from these considerations, the present paper aims to answer the following research question: can (a generalisation of) Gibrat's law allow us to infer Zipf's law and vice versa, by empirically analyzing the link between these two laws, in the context of urban growth, and, in particular, the dynamics of city size distributions? In this framework, particular attention will be paid to the role of the mean and variance of the city population as a key indicator for assessing the validity (or non-validity) of the generalised Gibrat's law.

Consistently with Eeckhout (2004), we focus our empirical investigation on the entire city size distributions of five selected countries (Botswana, Germany, Hungary, Luxembourg and Malta) and not only on the upper tail,[2] as other studies have done (see among others, Giesen and Suedekum, 2011; Guerin-Pace, 1995; Rosen and Resnick, 1980 and Soo, 2005). First, we are able to find evidence of the existence of Gibrat's Law for three out of the five preselected countries; we then test the empirical relationship between the (generalised) Gibrat's law and Zipf's law, by considering the dynamics of the hierarchical structure of the various city systems, on the basis of the mean and variance indicators.

The paper is then organised as follows. In this section (Section 1), we first summarise Gibrat's law (Subsection 1.2) and Zipf's law (Subsection 1.3); we then address the literature on the relationship between these two laws (Subsection 1.4). This short review constitutes the basis for constructing our empirical analysis aimed at testing Gibrat's law vs Zipf's law in specific case studies. Section 2 describes the rationale underlying the selection of the five countries under analysis (Subsection 2.1), by focusing on their different spatial economic characteristics and related statistics (Subsection 2.3), while subsequent sections illustrate the results of the empirical analysis devoted to testing Gibrat's law (Section 3), as well as the link between Gibrat's law and Zipf's law (Section 4). The paper concludes with some methodological considerations and directions for future research (Section 5).

1.2. Gibrat's Law

In 1931, Gibrat observed that the growth rate of a city's population does not depend on the size of the city. In other words, although cities can grow at different rates, no systematic behaviour exists between their growth and their size, so that, according to Gibrat (1931), we cannot affirm that larger cities grow faster than smaller ones or *vice versa*. More formally, we can say that a log-normal distribution (or Gibrat distribution) arises (if certain conditions hold) "*as the limiting distribution of the product of*

2 Eeckhout (2004) shows that if the city growth does not depend on city size "*then the estimated OLS coefficient of the so-called rank-size rule varies depending on the truncation city size, i.e. the inclusion of smaller (larger) cities in the sample, leads to a smaller (larger) coefficient*" (Fazio and Modica, 2012, p. 3).

positive random variates and the number of terms in the product tends to infinity" (Chesher, 1979, p. 403). Analytically, we can write the following logarithmic expression, as in Steindl (1968):

$$\log P(t) = \log P(0) + \varepsilon(1) + \varepsilon(2) + \mathrm{K} + \varepsilon(t) \tag{1}$$

where $P(t)$ is the size of a certain city at time t, $P(0)$ is the initial population, and $\varepsilon(t)$ is a random variable (indicating random shocks), i.i.d random variable with mean μ and variance σ^2. Equation (1) identifies the logarithm of the size of a given city as the sum of the initial size and past growth rates. It should be noted that this stochastic process leads to the log-normal distribution (1) of the variable $P(t)$, only if a sufficiently strong condition holds, namely, the law of proportionate effect. This law can now be interpreted as follows: "*A variate subject to a process of change is said to obey the law of proportionate effect if the change in the variate at any step of the process is a random proportion of the previous value of the variate*" (Chesher, 1979, p. 403). The implication of Gibrat's law is that the growth processes of cities have "*a common mean (equal to the mean city growth rate) and a common variance*" (Gabaix, 1999, p. 741), that is both the mean and variance have to be independent from the size of the cities.

In economic terms, the above formula means that, if we impose noise on a process, after a certain (long) time, the deterministic pattern generated by some fundamental structural variables appears again. This is important, if we want to build models close to empirics; and hence: "*economic models that explain city size distribution by relying on characteristics of hierarchies between cities, demand supply curves, technological considerations, and the like are at best incomplete if they fail to satisfy Gibrat's law in the end*" (Gabaix, 1999, p. 742).

However, Gibrat's law is only a part of the story. Indeed, this law is related to the rank-size rule, and, in particular, to Zipf's law. The link between the two regularities has been widely debated, because the proportionate growth rate process (Gibrat's law) gives rise, as Gibrat (1931) stated, to the log-normal distribution (see (1)); the proportionality of the size to the rank (Zipf's law, or the more general rank-size rule) is associated, instead, with a Pareto[3] distribution. Yet many studies have shown that a random growth process can generate power laws[4] (see Richardson, 1973, for a review).

In the scientific literature, the link between a log-normal and Pareto distribution is still a matter of controversy. According to several authors, the Gibrat (log-normal) process can generate Zipf's law only at the upper tail distribution (see e.g. Blank and Solomon, 2000; Eeckhout, 2004, 2009; Levy, 2009), as

3 Note that the slope coefficient of the rank-size rule represents the inverse form of the parameter of the conventional Pareto distribution. See, for more details, Adamic (2007) and Parr (1985).

4 The Zipf distribution is a particular distribution of the power law family.

already outlined by Parr and Suzuki in 1973 (p. 343): "*truncation of the log-normal at an appropriately high level enables the truncated portion to be regarded as not significantly different from the rank-size distribution*". Furthermore: "*these distributions all belong to the same family and in the upper tail are similar and consistent with the rank-size rule. Lower parts of these distributions, however, often exhibit significant differences and frequently do not conform to the rank-size rule*" (Carrol, 1982, p. 5). However, Gabaix (1999) has shown that the presence of an infinitesimally small barrier, on an identical growth process across sizes, necessarily leads to a Zipf distribution (see also Subsection 1.4 below). Against this methodological background, we briefly review in the next section the formal definition of Zipf's law and its spatial economic interpretations.

1.3. Zipf's Law

The second well-known spatial regularity is given by the so-called Zipf's law (on the basis of a first study by Auerbach[5] in 1913). In 1949, Zipf observed and established that the sizes of the cities in a country are proportional to their rank. This means that for example, in Botswana, the size of Gaborone is roughly twice the size of Francistown, the second largest city, three times the third largest city, Molopolole, and so on. Formally, this can be written as:

$$P_i = KR_i^{-q} \tag{2}$$

Equation (2) is known as the rank-size rule and is usually expressed in logarithmic form, as follows:

$$\log(P_i) = \log(K) - q\log(R_i) \tag{3}$$

where P_i is the population of city i, R_i is the rank of the ith-city and K is a constant. Zipf's law holds precisely, when the coefficient q is equal to one. Several interpretations of the Zipf coefficient, q, have been proposed in the literature. In principle, the q-coefficient can be seen as an indicator of the hierarchical degree of a system of cities (Singer, 1930). In fact the q-coefficient measures how unequal the city distribution is: the higher the q-coefficient, the more unequally distributed is the city system. On the contrary, the smaller the value of q, the more even is the system of cities (in the extreme, when q=0, we have a very even system of cities all of the same size; when $q=\infty$, instead, we have only one

5 "*The population of a city is inversely proportional to the number indicating its rank among the cities of a given country*" (Auerbach, 1915, p. 384).

city hosting the entire population). Related to this interpretation, Singer (1930) considers this q-coefficient as an index of metropolisation; in particular, this author affirms that the higher the q-coefficient, the more important is the urban land value. In the same vein, Parr (1985) shows the existence of a U-shaped pattern of the Pareto coefficient (the counterpart of the q-coefficient) over time; he establishes a link between the overall level of development and the degree of metropolisation, that is, the more developed a country, the greater will be the degree of metropolisation. Interestingly, Parr (1985, p. 208) argues: "*the process of concentration is facilitated by (and ultimately dependent on) improved interurban and interregional transportation*". Against this background, Brakman et al. (2001) interpret q as an indicator of industrialisation and agglomeration economies, while more recently, Reggiani and Nijkamp (2012) have considered the urban structure in a country as a socio-economic connected network and interpret q as an indicator of economic development by comparing it with the connectivity degree distribution[6] in a network.[7]

In summary, Gibrat's law expresses the growth process of a certain variable (firm, city, income, wealth, etc.), independent of its size, while Zipf's law presents the static relationship of the size of this variable with its rank. The main concern in the literature has been whether and how Gibrat's law and Zipf's law are mutually linked, for example, by means of the same formal model of the size distribution of the variable in hand, within a given urban-spatial system. In the next section, we briefly outline the main methodological challenges which constitute the basis for our subsequent empirical analysis.

1.4. The Relationship between Gibrat's Law and Zipf's Law

"*It has long been noted, in economics since at least Champernowne (1953),..., that random growth process could generate power laws*" (Gabaix, 1999, p. 741). The debate on the two distributions is still very alive, mainly because it is a difficult task to identify the most discriminating between these two, at least in the upper tail of the distribution (Perline, 2005). As mentioned in Subsection 1.1, both are similar and consistent with the rank-size rule in the upper tail (Carrol, 1982).

Many arguments have been put forward in the literature for explaining distinct variations between these distributions. Here, we focus on stochastic models, "*perhaps the most influential theories bearing upon the rank-size problem*" (Carrol, 1982, p. 5). The stochastic approach mathematically derives certain

6 "*The degree distribution, P(k), gives the probability that a selected node has exactly k links*" (Barabasi and Oltvai, 2004, p.102)

7 In particular, Reggiani and Nijkamp (2012) show the following: high values of the connectivity degree distribution (>3) match a random network, which corresponds to a homogeneous urban setting, characterized by low values of q (<0.5). Vice versa, small values of the connectivity degree distribution (<0.5) indicate a hub configuration in the network, which corresponds to a high urban heterogeneity, expressed by a value of q >1.

distributions of city size by starting from a given stochastic process; more precisely, these models postulate that different distributions arise as the result of steady state outcomes of different stochastic processes describing the underlying economic forces (Steindl, 1968). In particular, these models postulate that rank-size regularities are the steady-state equilibria of the law of proportionate effect, namely Gibrat's law. Then proportional growth can explain the Pareto/Zipf distribution. In this section, we review the main models which have influenced subsequent studies; for an extensive review, the reader can consult Carrol (1982) and Suarez-Villa (1988).

Champernowne (1953) presents a model on income distribution (this can be adapted to city size as well) based on the stochastic process from Markov chains.[8] The model takes into consideration a stochastic matrix, namely, a matrix where all probabilities of transition from a class of income to another are reported. Furthermore, the model makes the assumption of a constant number of incomes through time: "*Under these circumstances, and provided certain other conditions*[9] *are satisfied, the distribution will tend towards a unique equilibrium distribution dependent upon the stochastic matrix but not on the initial size*" (Champernowne, 1953, p. 318). This model is able to explain the Pareto distribution for income size. Simon (1955) proposes instead an interesting study on the class of skew distribution functions. He shows how a preferential attachment process, that is a law of proportionate effect with a constant probability that new small units enter at any time in the process, leads to a class of skewed distributions, incorporating among them both the Pareto and log-normal distribution. Gabaix (1999, p. 750) presents a model with a random walk with a small barrier "*and, more interestingly, that, as the barrier become lower, the exponent (of the Pareto distribution) converges to 1. So, in essence, all we need is an infinitesimally small barrier, to ensure that the steady state distribution will be Zipf*". Thus, Gabaix's model shows that a proportional growth process can lead to an exact Zipf distribution.

All of the above mentioned studies have provided one important contribution to the literature: under plausible conditions,[10] a proportionate growth process can lead to a Pareto/Zipf distribution. In summary, Champernowne's model (1953) established that a proportional growth process leads to the limit of a Pareto distribution. Simon's model (1955) generalises Champernowne's results, proving that Gibrat's law can lead to different skewed distributions, one of which is Pareto. Finally, Gabaix (1999) shows that Gibrat's law can lead to an exact Zipf distribution (see also Cordoba, 2003). However, despite the extensive literature on Gibrat's law, according to Kalecki (1945), the model proposed by

8 Recall that Gibrat's law can be interpreted as a Markov process.

9 In particular, the stability condition used in this model implies a quite unrealistic negative expected value of a change in income (Steindl, 1968).

10 In particular, a) negative expected change of income in Champrnowone (1955); b) steady inflow of new and small cities in Simon (1955); and c) lower barrier for Gabaix (1999).

Gibrat, although formally correct, has implications that are not very realistic, particularly in relation to the variance.[11]

Kalecki (1945) assumes that the variance of the random variable $\varepsilon(t)$ in Eq. (1) changes over time for three reasons: (i) the variance may change due to economic forces only; (ii) the variance may increase purely due to the influence of random shocks, and (iii) the variance may change due to both economic forces and random shocks. He thus proposes a model in which there is a negative correlation between the size P and the random variable $\varepsilon(t)$ (in Eq. (1)). In this way, as size increases (as time grows), the random shocks are smaller, thus preventing the tendency of the variance to increase. In this context, Cordoba (2008, p. 1463) proposes a: "*generalisation of Gibrat's law that allows size to affect the variance of the growth process but not its mean*". In particular, one of the implications of Cordoba's generalised model is that non-proportionality of the variance is required to take into account a q-coefficient different from one (in Eq. (3)). More specifically, the larger the q-coefficient, the more unequal is the distribution, and this makes a growth process more volatile.[12] On the basis of Cordoba's results, we can outline the following relationships between Zipf's law and Gibrat's law:

(a) If q=1, Zipf's law holds. In order that Gibrat's law applies, neither the mean nor the variance of growth can depend on size.

(b) If q>1, the distribution is more unequal. In order that Gibrat's law applies, it is necessary that the mean is independent of the city size, but not the variance; indeed, the associated growth process requires that smaller cities face a greater volatility of growth than larger cities.

(c) If q<1, the distribution is more evenly distributed. Again, in order that Gibrat's law applies, it is necessary that the mean is independent of the city size, but not the variance. Here, the associated growth process requires that larger cities face a greater volatility of growth than smaller cities.

Starting from these considerations, the main challenge and aim of this paper is to empirically explore the above mentioned relationship "Gibrat's law vs Zipf's law", according to the three statements (a)-(c) above. It should be noted that empirical investigations of the relationship "Gibrat's law vs Zipf's law" are still rare. We may refer here, amongst others, to studies on firm growth (see Fujiwara et al. 2003), and studies on city growth (Berry and Okulicz-Kozarin 2012; Dittmar 2011; Giesen and Suedekum 2011). All of these analyses have mainly focused on the first item, (a), the relationship between Zipf

11 The law of proportionate effect, indeed, assumes that a given variable $P(t)$ changes by a random and independent effect amount as time goes by; so even though it is realistic that the mean remains constant as t increases, the same is not true for the variance because it should increase instead.

12 The volatility is a measure of fluctuation of a process. We will use the variance as an indicator of the volatility of an underlying proportionate growth process.

and Gibrat. To the best of our knowledge, no empirical application has focused, so far, on how (partial) deviations from either law can affect the other law; in other words, on statements (b) and (c).

Given these propositions, we aim to test items (a)-(c) in the context of urban dynamics, by examining urban patterns in countries which exhibit different spatial-economic characteristics. Our methodology includes two main steps: (i) we first analyse Gibrat's law for the set of cities in the selected countries (Section 3), in order to (ii) construct a validation framework to assess the relationship "Gibrat's vs Zipf's law"(Section 4). In Section 2 which follows, we will first describe the characteristics of our case studies, in particular the countries to be investigated.

2. Choice of Case Studies: Descriptive Analysis and Statistics

2.1. Preface

Given our empirical objective, aiming to analyse Gibrat's vs Zipf's law in different spatial socio-economic landscapes, the rationale underlying the choice of our case studies is the following. We have selected five distinct countries characterised by different typologies, according to five main distinguishing criteria: (a) OECD vs non-OECD country; (b) advanced economy vs non-advanced economy; (c) centrally located vs non-centrally located; (d) size of surface; and (e) increasing vs decreasing population. On this basis, we have identified five countries: Botswana, Germany, Hungary, Luxembourg and Malta for study.[13] In Table 1 we report, for each country, some economic indicators (such a GDP per capita, growth rate and percentage of investment over GDP), as well as some other important indicators for the mobility and transportation system (such as the length of railways and roadways, and the number of cars per thousand people).

Some points are worth noting here. Botswana is the only non-OECD country, while all the others are OECD countries. Botswana shows the features of a non-advanced[14] economy; however, it exhibits a trend towards an increase in population and economic growth. Germany was a founding member of the European Community in 1957 (which became the European Union (EU) in 1993); it is central in Europe and is a large country in terms of surface area and population, with an advanced economy. Hungary joined the EU in 2004; it is located in central Europe, but shows a non-advanced economy and a decreasing population. Luxembourg, like Germany, was a founding member of the European Community in 1957; it is a small country, but very central in Europe with a high income per capita. Malta, like Hungary, joined the EU in 2004; it is a small, peripheral country but with an advanced

13 Alphabetic order.
14 According to the IMF classification.

economy; it is also one of the most densely populated countries worldwide. Clearly, other choices could have been made, but the present set of countries should represent a sufficiently interesting collection of cases for in-depth investigation.

We will now concisely outline the major economic and demographic characteristics of these countries (Subsection 2.2). We will then show some descriptive statistics (Subsection 2.3) and, finally, we will offer a descriptive analysis (Subsection 2.4).

2.2. Profiles of Case Studies

In this section we will describe a few characteristic features of the urban system of five case countries further analysed in our study.

Botswana is the largest country in the sample in terms of size: it is around 581,000 sq. km, but with a very low population density: 3.19 inhabitants per sq. km. Furthermore, it is poor in terms of transport infrastructures and of number of cars per 1,000 people; the urban population covers only 61% of the entire population. On the contrary, it is a country with high expenditures in investment, and with a large annual growth.

Germany, instead, is the most developed country from an economic point of view. It has the largest population in Europe and is also large in terms of surface area, 357,000 sq. km. Moreover, it is the second densest country in our case studies. It also has very good infrastructures.

Hungary is a mid-sized country with a low degree of transport infrastructures and a low average number of cars per inhabitant. The population density is low and also the number of urbanised people is low.

Luxembourg is a small but rich country, highly connected with the rest of Europe. It has one of the largest ratios of number of cars to population size. In terms of GDP per capita it is the richest country in our sample.

Finally, *Malta* is the smallest country of our data-set in terms of both population and surface area; however, the population density is huge and this is reflected in the number of urbanised people.

< Table 1. About Here >

2.3. Data and Descriptive Statistics

We collected data from the National Institute of Statistics for all five of these countries.[15] In particular, we collected data from the Central Statistics Office of Botswana, the Institute for Employment Research[16] (IAB) in Germany), the Hungarian Central Statistical Office, the STATEC-Institute National de la Statistique et des Etudes Economique of Luxembourg, and the National Statistique Office of Malta.

It should be noted that an extensive debate concerns the type of unit under analysis: several studies have been carried out using metropolitan areas, i.e. by considering the entire population in a given city, as well as all populations of suburban areas. Nevertheless, here we aim to carry out an analysis as comparable as possible between the five countries, by also including, as much as possible, all the cities in a given country. For these reasons, we consider in our analysis the entities legally defined as cities or villages in their countries, although we are aware that the administrative definition given by legal borders might not fulfill our scopes exactly. In order to have a comparable unit, in all countries we have selected those localities which are similar to a municipality.[17]

Another concern is due to the fact that we have different temporal horizons, which, sometimes, are short. This is the case for Botswana, where we have only two census observations (2000 and 2010), as well as for Hungary, where, although the time span is 30 years (1980-2011), we have only four census observations. For Germany, however, although the time span is 15 years, we have annual data (1993-2007), so we can conduct a more precise analysis. Finally, Luxembourg and Malta have a long time series considering all census data from 1821 until 2011 for Luxembourg, and from 1901 to 2009 for Malta; unfortunately the time span between two subsequent observations is not constant even in the same country and especially before World War II.

In Table 2 we report some descriptive statistics. First, it is interesting to notice that the median for the five different countries reflects the percentage of urban people in any single country: Malta shows the largest median, 3,983 inhabitants, and we recall that the urbanisation in this country is over 90%; Luxembourg has a median of 2,018 people and an urban population of over 80%. Hungary shows the smallest median, 812 people, with a relatively low urbanisation rate, 68%.

Secondly, we report the (log) mean. In all five countries the mean is greater than the median which indicates asymmetry. The asymmetry of the distribution is also confirmed by the skewness. In general,

15 For all countries we have data over all cities from the biggest to the smallest one.

16 The authors wish to thank Uwe Blien and Anette Haas (IAB, Germany), for kindly providing the data used in our study on German cities (Sections 3 and 4).

17 We encountered some difficultly in making the right choice for Botswana where we also had data for small localities but we chose to collect all localities with ID code 100, namely villages and cities.

in city size probability distributions, we expect a positive skewness, which denotes the fact that most of the observations lie on the left of the distribution. As a consequence, the left tail is longer: this means that we expect a greater number of small cities than large ones. In our sample this is true for four countries: Botswana, Germany, Hungary and Luxembourg, while Malta shows a negative skewness,[18] thus indicating a greater concentration of large cities. The reason for this is likely due to the huge urbanisation rate in that country that pushes the concentration of people into large centres and this is magnified by the small size of Malta.

Kurtosis is a measure of peakedness of the distribution. In all our case studies, the value of kurtosis is positive: this indicates a situation in which the distributions show heavy tails and peakedness with reference to a normal distribution (whereas negative kurtosis indicates light tails and flatness). We notice the presence of fat tails and peakedness especially in Botswana (which shows the highest positive kurtosis[19]).

< Table 2. About Here >

Following on from the above observations, in Section 3 we focus our attention on the validity of Gibrat's law, in order to design an analytical framework that is useful for meeting the ultimate goal of our analysis: a comparison of Gibrat's' law and Zipf's law.

3. Testing Gibrat's' Law: Method and Results

3.1. Preface

Given our objective first to test the validity of Gibrat's law in the five countries concerned (Botswana, Germany, Hungary, Luxembourg and Malta), in this section we show the adopted methods and the related results. In particular, we consider two different methodologies: (i) parametric analysis; and (ii) non-parametric analysis. Concerning the parametric technique, we search for deviations of sizes from their mean, first in a cross-sectional setting (Subsection 3.2.1) and then in a longitudinal setting (Subsection 3.2.2). Concerning the non-parametric technique, we look at the mean and variance of the

18 Negative skewness might be seen as a contradiction with a mean greater than the median, but this is not the case. For example, von Hippel (2005) argues: "*the mean is right of the median under right skew, and left of the median under left skew. This rule fails with surprising frequency... It can fail in distributions where one tail is long but the other is heavy. Most commonly, the rule fails in discrete distributions where the areas to the left and right of the median are not equal*". Our case covers both.

19 De Carlo (1997, p. 294), in an interesting note on kurtosis, affirms: "*it represents a movement of mass that does not affect the variance. Consider the case of positive kurtosis, where heavier tails are often accompanied by a higher peak. Note that if mass is simply moved from the shoulders of a distribution to its tails, then the variance will also be larger. To leave the variance unchanged, one must also move mass from the shoulders to the centre, which gives a compensating decrease in the variance and a peak*".

size, given the initial size (Subsection 3.3).

3.2. Parametric Analysis

3.2.1. Model A: OLS Regression

In this section we will use an OLS regression model and report the results from the parametric analysis. We check dynamic deviations from the proportionality of mean growth and variance to size, by using a method firstly proposed by Kalecki (1945) and subsequently utilised, among others, by Bottazzi et al. (2001). In particular, the adopted model is the following OLS model:

$$g_t^i = \beta_t g_{t-1}^i + \varepsilon_t^i \tag{4}$$

where $g_i(t)$ is the deviation of the logarithm of the population of city i from the mean of the logarithms of the city populations at time t, and ε is the error term. β is the parameter to be estimated and "*provides an estimate of the divergence/convergence of the size distribution toward its mean*" (Bottazzi et al., 2001, p. 1184). Gibrat's law holds if β is equal to one.[20] When β is lower than one, this means that size converges towards its mean, namely, the larger a city, the smaller the expected growth. On the contrary, when β is greater than one, the larger a city and the larger the expected growth. We test the model for each time-step.

As an indicator of the volatility of the growth process, we use the variance ratio, θ_t, between the variance of g at time t, $\sigma(g_t)$, and the variance of g at time t-1, $\sigma(g_{t-1})$:

$$\theta_t = \frac{\sigma^2(g_t^i)}{\sigma^2(g_{t-1}^i)} \tag{5}$$

If the variance is stable along two subsequent years, the variance ratio, will be close to unity.

< Table 3a. About Here >

< Table 3b. About Here >

Two main conclusions arise from here. Firstly, looking at the parameter β, Gibrat's law does not always hold (over time). Germany, Luxembourg and Malta (especially the latter two) are clear examples of this

20 We report here only the condition on the estimated β. However it should be noted that another condition is necessary to affirm that Gibrat's law is in operation, indeed the error terms have to be serially uncorrelated. We, then, add one more lag in Eq. (4) to very this additional condition. In most of the cases the error terms result serially uncorrelated.

intermittency: we can see periods where Gibrat's law holds and others where it does not. Secondly, it seems that the effect of the (non)validity of the law is lengthy: namely, Gibrat's law in general holds (or does not hold) continuously for two or three time windows. Consequently, the first result of our analysis is that testing Gibrat's law requires a data-set of considerable length, or as many possible observations as we can. It is also interesting to note that in general, when Gibrat's law does not apply, the variance at time t is higher than the variance at the previous times; this is denoted by the parameter θ in Eq. (6) greater than one. This is, of course, consistent with the idea of Gabaix (1999) which, according to Gibrat's law, both the mean and variance of the growth rate have to be independent with respect to the size.

In more detail, by observing β in Tables 3a and 3b, we can see that in Botswana, Luxembourg and Malta, Gibrat's law holds quite often (β=1). In Germany it does not apply more than half of the time and in Hungary Gibrat's law never holds.

This analysis, although intuitive, presents some shortcomings due to the fact that it does not allow all the temporal aspects to be included. In particular, even though it is good to have yearly observations, some doubt about the validity of the above method can arise when the time windows are longer. For this reason, as previously anticipated, we utilise a further parametric technique (model B), firstly suggested by Clark and Stabler (1991) and adopted, among others, by Black and Henderson (2003). The emerging results are illustrated in the next section.

3.2.2. Model B: The Unit Root Test Approach

The starting point of the second, complementary parametric method is that testing for proportional growth implies testing for a unit root process. In other words, we are looking for the presence of mean reversion in the stochastic growth process. Mean reversion is a mathematical concept denoting that a process of both high and low growth is temporarily present and that population growth will tend to move to the average growth over time. Following Black and Henderson (2003) we use:

$$\ln(P_t^i) = \alpha + \delta_{t-1} + \gamma \ln(P_{t-1}^i) + v_t^i \tag{7}$$

where, α is a constant, δ_{t-1} are fixed time effects, P^i is the population of city i at time t and $t-1$. γ is the parameter to be estimated and if Gibrat's law holds, it implies that γ=1.[21] Black and Henderson (2003) argue that this null hypothesis does not permit an auto-regressive process to the error so pooling OLS

21 A value of γ lower than one implies mean reversion.

suffices.

< Table 4. About Here >

The results are summarised in Table 4. In general Gibrat's law holds for those countries where Gibrat applies cross-sectionally more than a half time:[22] Botswana, Luxembourg and Malta. In Germany, although the estimated parameter, γ, is very close to unity, it is significantly different from one because of the very small standard error. Finally, in this case, Hungary does not indicate the presence of Gibrat's law.

In summary, according to both parametric methods A and B, the urban dynamics of Botswana, Luxembourg and Malta seem to capture Gibrat's law, while Gibrat's law does not appear to be appropriate for the urban evolution underlying Germany and Hungary.

Given this preliminary analysis of Gibrat's law, based on parametric methods, we explore the validity of these first results, by means of a non-parametric analysis. Non-parametric analysis provides an important tool to explore directly the independence of the mean and variance of the growth from the size. In this way we can gain an indication of the behaviour of the mean and variance.

3.3 Non-Parametric Analysis

In this section we show the results of the non-parametric analysis strictly based on Ioannides and Overman (2003). We use the normalised growth rate, namely the difference between a city's growth rate and the mean city growth rate, all divided by the standard deviation of growth. The strength of non-parametric estimation is that we do not impose any relationship between the dependent and independent variables. According to Cameron and Trivedi (2005, p. 294), we: "*let the data show the shape of the relationship*"*;* this is an especially convenient approach when we do not know *a priori* the correct distribution of the data. In our analysis, we will use the Nadaraya-Watson (NW) method (Nadarya 1964; Watson 1964), where the bandwidths are calculated with an optimal rule of thumb.[23]

If Gibrat's law holds, the non-parametric estimation of the conditional mean and variance should be stable across different population sizes. Furthermore, because of normalisation, we expect the conditional mean growth to be equal to zero, and the conditional variance of growth equal to one. It should be noted that, while the standard parametric regression methods provide only an aggregate relationship between growth and size which is constrained to hold over the entire distribution of city

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22 See Tables 4a and 4b.

23 We refer readers to their papers for a more detailed description.

sizes, the non-parametric estimates allow the growth to vary with size over the distribution.

< Figure 1 About Here >

In Figure 1 we show the NW estimator for conditional mean growth (upper panel) and variance (lower panel) for the entire city size distribution. Following Cordoba (2003), the independence of the expected conditional growth rate always has to be satisfied, while the variance can be affected by the city size. In general, smaller cities face a faster growth than larger ones. However, very quickly (in most cases), the conditional mean appears to become stable. This evidence is consistent with the model of Gabaix (1999) where a truncation concerning the small cities is necessary to have stationarity.

By considering the specific countries in Figure 1, Luxembourg and Malta show a striking stationarity in mean and variance, so that we can affirm that in these cases, Gibrat's law holds. Botswana appears to face a huge variability at the bottom of the distribution for both mean and variance, but in the upper tail the evidence of independence of the mean is striking too. Thus, in principle, we can also accept Gibrat's law for this country. We can, on the contrary, reject Gibrat's law for Germany and Hungary, by confirming the results emerging from the parametric analysis (see Subsection 3.2).

This non-parametric test suffers from several shortcomings: firstly, outliers have a huge impact on the results, in particular the variance (Eeckhout, 2004; Gonzalez-Val et al., 2012); and secondly, we cannot directly compare the q-coefficient with Gibrat's law.

For these reasons, after the previous tests that confirm Gibrat's law for Botswana, Luxembourg and Malta, the final methodological step is the analysis of the relationship between Gibrat's law and rank-size/Zipf's law, by considering, as key indicators, the dynamics of the q-coefficient, the β-parameter and the θ-parameter; in other words, the evolution of the dynamics of economic development of the country. This will be reported in the next section.

4. Gibrat's Law and Zipf's Law: A Comparative Study

4.1 Role of the Adopted Parameters

In the previous sections we have shown that Botswana, Luxembourg and Malta seem to obey Gibrat's law, while this seems not to be the case for Germany and Hungary. The final step in our analysis is then the investigation of the relationship between Gibrat's law and the rank-size/Zipf's law, by means of the rules (a), (b) and (c) (outlined in Subsection 1.4).

From the operational viewpoint, we investigate the relationship between the q-coefficient in Eq. (3) and the estimated parameters β and θ from Eqs. (4) and (5), on the basis of Cordoba's propositions (a); (b) and (c) (Subsection 1.4). In particular, we estimate the q-coefficients in the rank-size rule (3) by means of a modification proposed by Gabaix and Ibragimov (2011), according the following:

$$\log(P_i) = \log(K) - q\log(R_i - 0.5) \tag{8}$$

where P_i, K, q and R_i are the same as in Eq. (3).

In Tables 5a and 5b we report the estimated q-coefficients and the parameters β and θ, according to Eqs. (8), (4) and (5), respectively, for each of the five countries.

Concerning the coefficient q (Eq. (8)), it should be noted that we interpret the q-coefficient as a measure of hierarchy of city size distribution. In this sense a positive change in the estimated q-coefficient denotes a situation where larger cities have grown more than smaller ones (in relative terms); thus an increasing q-coefficient (see Eq. (3)) reflects the tendency towards agglomeration economies in the country at hand (see Subsection 1.4).

It is interesting to also pay attention to the β-parameter (see Eq. (4)), which is the degree of divergence of the size distribution from its mean: a β-value lower than one indicates that larger cities have an expected growth lower than smaller ones. We note here that the condition β=1 indicates the validity of Gibrat's law (Bottazzi et al. 2001).

Finally, the parameter θ reflects the ratio between variance at time t and t-1 (see Eq. (6)). This provides a measure of the volatility of the growth process as a θ-value equal to one indicates the stability of the variance between years.

Overall, by means of these three parameters, we can experiment with the propositions (a), (b) and (c) in Subsection 1.4. For example, an increasing/decreasing q-coefficient – indicating changes in the growth rate between large and small cities – should lead to a generalised Gibrat's law. It appears then that the q-coefficients, together with the β- and θ-parameters, offer insights into different aspects of the same growth process: the q-coefficient captures the output of the growth process, while the β- and θ-parameters take into account the mean and variance of the growth process, respectively. In the latter context (regarding the role of the mean and variance), it seems worthwhile to test the different dynamics of the large cities vs the small cities, in order to explore in more detail where a greater volatility shows up. We can then split, for each country, our sample into two halves by defining two sub-samples; one for the large cities and the other one for the smaller cities. We then estimate the parameters β and θ, according to Eqs. (4) and (5) respectively, for these two sub-samples. In this way we can analyse, firstly, whether Gibrat's law holds separately for large and small cities; and, secondly, whether the growth process is more volatile.

We recall that, according to Cordoba (2003), in order to preserve a Pareto/Zipf coefficient different from one, the underlying growth process has to be different for smaller and larger cities (Subsection 1.4).

< Table 5a. About Here >

< Table 5b. About Here >

In the next sections, the role of the various parameters q, β and θ in capturing the relationship "rank-size rule vs Gibrat's law" will be illustrated with reference to the empirical analyses in each of the five countries.

4.2 Botswana

Starting with Botswana, we can see that the estimated q-coefficient is greater than one for both the years 2001 and 2011, indicating a predominance of larger cities. In particular, in 2011 the estimated q-coefficient is slightly greater than that one in 2001, thus showing a tendency – in the last decade – towards a higher economic development. By considering the relationship with Gibrat's law, we then investigate condition b) of Subsection 1.4. Considering the entire sample, we have already shown that Gibrat's law holds in 2011 with an estimated β-parameter not significantly different from one (β=0.995**).[24] However, considering the two sub-samples, we find evidence of Gibrat's law for large cities (β_{large}=0.979**) but not for small ones (β_{small}=0.761). This indicates that larger cities of the sub-sample of small cities (i.e. medium size cities) have an expected growth lower than smaller ones.[25] Thus, large and small cities face two different underlying growth processes; however, this is still consistent with proposition b) of Subsection 1.4 predicting that the associated growth process requires that smaller cities face a greater volatility of growth than larger cities. For this reason we now analyse the behaviour of the variance.

The variance ratio for the entire sample in Botswana is greater than one (θ=1.078), indicating a greater volatility of the process in 2011. This latter condition is not enough to investigate our statements b) of Section 1, because it only refers to the "temporal" non-stability of the variance, without considering the

24 Where ** indicates a significance level at 5%.

25 This fact confirms the results in Section 3.3 in Figure 1: our estimated conditional mean growth is below zero in the mid-range of cities (city size in logarithmic terms between 6 and 8). We recall that if Gibrat's law holds, the non-parametric estimator in Section 3.3 should be constant across sizes and equal to zero. The last implication is due to the effect of the normalisation used (Ioannides and Overman, 2003)

"spatial aspect", namely the (non)independence of the variance with respect to the size of the cities.[26] For this reason, we analyse the two sub-samples separately, as previously anticipated. The variance, θ, for large cities shows a striking stability (θ_{large}=1), while, for the small cities, it is slightly greater than one (θ_{small}=1.011), implying an (increasing) change in the underlying volatility of the growth process for small cities. Given this fact, we can affirm that at time t (2011), the variance is unchanged for large cities but increases for the small ones, indicating a dependence of variance with respect to size; in particular, smaller cities face a greater volatility than large cities. In summary, statement b) (Subsection 1.4), which affirms: "if q>1, in order that Gibrat's law occurs, it is necessary that the mean is independent from the city size but not the variance, indeed the associate growth process requires that smaller cities face a greater volatility of growth than larger cities", is satisfied for the whole sample.

4.3 Germany

Germany shows an U-shaped q-coefficient: it decreases until 1999 and then it increases. In fact Germany shows a lower degree of agglomeration between 1993 and 1999, namely larger cities become less "heavy" in the city system. After 1999, Germany shows again a process of concentration indicated by the increasing q-coefficient. By considering the relationship with Gibrat's law, we then investigate condition b) of Subsection 3.1. This case is somewhat different from the previous case for one main reason: the q-coefficient also shows a decreasing trend. Considering the entire sample, we have five years in which Gibrat's law holds. In particular, in 1995, 1996, 1999, 2000 and 2003, the estimated β-parameters are not significantly different from one.

Now if we focus on the period 1993-1999, where a decreasing q-coefficient applies, we can note that β_{large} is significantly lower than one, whereas β_{small} is not significantly different from one in most cases, indicating a situation in which the larger the city, the lower the expected growth. On the contrary, in the period 2000-2007,[27] where an increasing q-coefficient applies, we can notice that β_{large} are often not significantly different from one, whereas β_{small} are (most of the time) significantly greater than one, indicating a situation in which the larger the city, the larger the expected growth. In this situation we can figure out the following growth processes: when q-coefficient is decreasing, we have modifications on the growth process of large cities; in particular, the larger the city, the lower the expected growth. On the other hand, when q is increasing, small cities present a different growth process, namely the

26 It suggests a change in the variance over the time and then this might also imply changes in the dependence of the variance over the size.

27 Notice that in the years in which the q-coefficient is equal to that of previous year (i.e. 1998, 1999 and 2000), Gibrat's law holds. The stability of the process should not lead to any changes in the hierarchical structure of cities.

larger the city, the larger the expected growth. However, note that both cases should lead to the same effect on the underlying growth process (i.e. a greater volatility of the variance for small cities), in order to satisfy condition b). For this reason, we analyse the variance ratio, θ. By considering the entire sample, the variance ratio, θ, is often close to unity, but slightly lower than one until 1999 when it becomes stable (and equal to one). Again, this latter condition says few things about the independence of the variance from the size. We then analyse the two sub-samples separately and in particular we analyse those years where Gibrat's law holds (according to proposition b) of Subsection 1.4). Unfortunately, we do not have enough observations to make any inference about proposition b) in the period 2000-2007 because Gibrat's law holds only in 2003. Focusing on the period 1993-1999, it should be noted, firstly, that when Gibrat's law holds, the variance ratios for large cities, θ_{large}, are less than one, that is the variance at time t is lower than that at time $t-1$. Instead, the variance ratios for small cities, θ_{small}, are greater than one. At (any) time t, large cities face a lower volatility while small cities face a greater (or almost stable) volatility. We can then affirm that, at (any) time t, small cities face a greater volatility of growth. This is again consistent with proposition b) of Subsection 1.3. Again we can affirm that for those years in which Gibrat's law holds (β=1), statement b) (Subsection 1.4), which affirms: "if q>1, in order that Gibrat's law occurs, it is necessary that the mean is independent from the city size but not the variance. In particular the latter should be greater for small cities", is satisfied for the whole sample.

4.4 Hungary

In Section 3, we have shown that Gibrat's law does not hold in Hungary. We have already mentioned the role of the capital (Budapest) in this country, which attracts most of the population and most of the investment on infrastructure. In percentage terms, more than half of the total population of Hungary lives in the Budapest urban area. Moreover, it faces a migration flow from its rural area to the centre of the city. The evolution of the q-coefficient in this country reflects the tendency to agglomeration in the large cities: indeed the estimated q-coefficient is increasing and greater than one. In this situation we can check for statement b) of Subsection 1.4. Unfortunately Gibrat's law never holds, since the β-coefficients are always significantly greater than one. This means that size diverges towards its means, namely, the larger a city, the larger the expected growth. Then it is straightforward that the variance ratio, θ, increases for both large and small cities. However, it should be noted that the variance ratio of small cities, θ, is always greater than the variance ratio of large cities; thus, although we cannot formally show evidence of generalised Gibrat's law (in particular regarding statement b)), we again

show a greater volatility for small cities when $q>1$.
The analysis carried out over these three countries provides an important first conclusion. On the basis of statement b), we have shown that when $q>1$ and Gibrat's law holds ($\beta=1$), the variance ratio (θ-parameter) is actually greater for small cities. Moreover, when the q-coefficient is greater than one but decreasing, we have modifications on the growth process of large cities, but not on those of small cities: in particular, the larger the city, the lower the expected growth. On the other hand, when the q-coefficient is greater than one, but increasing, small cities present the opposite growth process: namely the larger the city, the larger the expected growth. Consequently, we find evidence of the generalised Gibrat's' law – as in statement b) of Subsection 1.4 – in the countries displaying $q>1$, where the independence of the mean with respect to the size is in operation, while the same is not true for the variance.
A reasonable criticism, at this point in the analysis, could be that generally, small entities (cities, firms and so on) present a greater volatility than larger ones. We can anticipate that we will find the opposite evidence in the case of $q<1$.
We now move to the situation where $q<1$. By considering the relationship with Gibrat's law, we investigate condition c) of Subsection 1.4 which predicts that the associate growth process requires that smaller cities face a lower volatility of growth than larger cities.

4.5 Luxembourg

Luxembourg shows an estimated q-coefficient lower than one. It increases until 1930, and after that it is not significantly different from one. Between 1821-1922, we are in condition c) of Subsection 1.4. Considering the entire sample, we have already shown that Gibrat's law holds in the first three years of the sample (1851-1880) and in 1922 with estimated β-parameters not significantly different from one. Moreover, considering the two sub-samples, most of the time we find evidence of Gibrat's law for both large and small cities. At a first glance it seems that large and small cities face the same underlying growth processes. However, to test statements c) of Subsection 1.3, we need to take into account the behaviour of the variance.
The variance ratio for the entire sample in Luxembourg in the period 1821-1922 is always greater than one, indicating a greater volatility of the process as time goes by. Indeed, when we split the sample in two halves, the variance ratios for the large cities show values always greater than one, while, for the small cities, they are always below one. This implies an (increasing) change in the underlying volatility of the growth process for large cities, in contrast to a (decreasing) change in the underlying volatility of

the growth process for small cities. Given this fact, we can affirm that at time t, the variance is increased for the large cities but decreased for the small cities, indicating a dependence of variance with respect to size; in particular, smaller cities face a lower volatility than large cities. In summary, statement c) (Subsection 1.4), which affirms: "if $q<1$, In order that Gibrat's law occurs, it is necessary that the mean is independent from the city size but not the variance, indeed the associate growth process requires that smaller cities face a lower volatility of growth than larger cities", is satisfied for the whole sample in 1821-1930.

In the period 1935-2011, however, the q-coefficient is not statistically different from one. By considering the relationship with Gibrat's law, we then investigate condition a) of Subsection 1.4 which predicts that the associated growth process requires that smaller cities face the same growth as larger cities. Considering the entire sample, we have already shown that Gibrat's law holds most of the time (estimated β-parameters not significantly different from one). Considering the two sub-samples, we find similar evidence for both large and small cities. However it is interesting to note that in those years where Gibrat's law does not hold, the estimated parameters β and θ show very different behaviour (i.e θ_{large}=.965 and θ_{small}=1.162 in 1970), but, in general, in those years where Gibrat's law holds, the differences between the estimators are not so large (i.e. θ_{large}=1.00 and θ_{small}=1.02 in 2004). In summary, statement a) (Subsection 1.4), which affirms: "if $q=1$, then in order that Gibrat's law occurs neither the mean nor the variance of growth can depend on size" is satisfied for the whole sample.

4.6 Malta

Finally, Malta presents a q-coefficient not significantly different from one. Again, by considering the relationship with Gibrat's law, we investigate condition a) of Subsection 1.4. Taking into account the entire sample, Gibrat's law holds most of the time (estimated β-parameters not significantly different from one). Considering the two sub-samples, Gibrat's law fails more in the large cities. In these cases, differences between the estimated parameters β and θ – in the two sub-samples – arise. Clearly, when Gibrat's law holds for both sub-samples, no major differences between the two parameters occur. In summary, statement a) (Subsection 1.4) is satisfied.

4.7 Synthesis

A synthesis of the above results – confirming the hypotheses by Cordoba (2003) – is presented in Table 6.

< Table 6. About Here >

The analysis carried out in this section prompts several interesting conclusions. We have been able to empirically verify the presence of a "generalised Gibrat law", as theoretically predicted by Cordoba (2003). In particular, we have verified statements a), b) and c) of Subsection 1.4. In more detail, we have shown that when $q>1$ (statement b)) and Gibrat's law holds ($\beta=1$), the variance-ratio (θ-parameter) is actually higher for the small cities, in comparison to that for large cities, indicating a larger volatility for small cities. On the contrary, when $q<1$ (statement c)) and Gibrat's law holds ($\beta = 1$), the variance-ratio (θ-parameter) is actually lower for the small cities, in comparison to that for large cities, indicating a larger volatility for the smaller ones. When $q=1$ (statement a)) and Gibrat's law holds, our findings agree with previous research, as both the mean and variance appear to be independent from the size.

Moreover, when the q-coefficient is greater than one but decreasing, we have modifications on the growth process of large cities, but not on those of small cities; in particular, the larger the city, the lower the expected growth. On the other hand, when $q>1$ but increasing, small cities present the opposite growth process, namely the larger the city, the larger the expected growth. We have, of course, an opposite behaviour when the q-coefficient is less than one.

5. Conclusion

The aim of our research work was to explore specific conditions leading to a generalisation of Gibrat's law in connection with the different typologies of rank-size distribution. For this purpose we empirically explored the link between the rank-size exponent, q, with the necessary conditions for Gibrat's law (that is mean and variance of the growth have to be independent from the size). We started our analysis based on the conclusion of Cordoba (2003, p. 3): "*Pareto distributions with larger exponents (more unequal distributions) require more volatile growth processes*". As far as we know, the conventional methodologies (Section 3) used to test Gibrat's law do not address this issue. In particular, a greater (lower) volatility of the variance is usually not empirically envisaged. We showed, instead, that, according to Cordoba (2003), the variance can be dependent on size if the rank-size coefficient is different from one; in particular, we verified what Cordoba (2003) calls a "generalised Gibrat's law" for different countries with different spatial-economic characteristics: Botswana, Germany, Hungary, Luxembourg and Malta. We found strong evidence of this generalised Gibrat's law for Botswana, Luxembourg and Malta. We found weak evidence of Gibrat's law for Germany and no evidence for Hungary.

Our results confirm the propositions provided by Cordoba (2003). In particular, when q=1, neither the mean nor the variance of growth depend on size; when q>1, the mean is independent of the city size, but not the variance, and small cities face a greater volatility in growth than larger cities; alternatively, when q<1, the mean is independent from the city size, but not the variance, and large cities face a greater volatility in growth than smaller ones. Gibrat and Zipf have offered complementary perspectives on city size and systems of cites in a given country. Their contributions are not necessarily identical, but offer new perspectives on the same multi-faceted prism of the space-economy. These laws are part of the same family, but also reveal specific distinct features. In particular, we find that Zipf's law and the rank-size rule behave like 'monozygotic' twins, while Gibrat's law seems to show the behavious of a 'heterozygotic' twin.

These results might be useful to 'relax' Gibrat's law in its strict interpretation, by reinforcing the hypothesis that small entities face a greater volatility in the growth process.

Our analysis prompts various intriguing research questions in the future. While Gibrat's law and Zipf's law mirror important organised structures in the topology of systems of cities, other relevant structural patterns may be investigated as well, such as the existence of fractal structures in urban systems (based, for example, on Mandelbrot's principles) or the persistent existence of spatial population or socio-economic disparities (based, for example, on Herfindahl's index). Clearly, the dynamics of such processes deserve due attention. In addition, the above applied investigation also calls for more fundamental research into the functional or behavioural backgrounds of such regularities. Three research directions are important here; (a) the interdependence between population indicators and broader socio-economic indicators for a system of cities; (b) the degree of various cities in the same national system; (c) the relationship between recent strong evolutionary trends in the digital world and the development of cities (and systems of cities).

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Table 1. Spatial Economic Characteristics of the Five Countries under Analysis

Country	Year	Km^2	Population	Density	% Urban pop.	% pop. growth	Car	Railway	Roadway	GDP per c.	% growth	Total Investment
Botswana	2011	581	1.85	3.19	62.00%	1.47	133	0.90	25.80	9,481	5.1%	21.52%
Germany	2007	357	81.78	229	74.00%	-0.20	564	41.90	644.50	40,403	2.7%	19.26%
Hungary	2011	93	9.99	107.4	69.00%	-0.18	347	8.10	197.50	13,045	1.70%	19.07%
Luxembourg	2011	2,5	0.51	205.6	85.00%	1.13	739	0.27	5.20	106,958	1.00%	21.17%
Malta	2009	0.32	0.42	1,338	94.00%	0.36	679	0	3.10	20,437	-2.70%	15.80%

Table 2. Descriptive Statistics of the Five Countries under Analysis

Country	Year	N. cities	ln(Mean)	ln (Variance)	ln (Median)	Skewness	Kurtosis
Botswana	2011	461	7.15	1.22	6.97	0.78	2.23
Germany	2007	12,262	7.42	1.50	7.30	0.34	0.17
Hungary	2011	3,154	6.75	1.34	6.70	0.40	0.95
Luxemburg	2011	116	7.81	0.92	7.61	0.86	1.45
Malta	2009	68	8.35	0.93	8.29	-0.59	0.31

Table 3a. Model A Estimates (Countries: Botswana, Germany, Hungary and Luxembourg; Different Years)

Country	Year	β	Robust s.e.	θ	R^2	N. obs.
Botswana	2011	.995**	.0149	1.0782	.92	460
Germany	1994	.999	.0003	1.0009	.99	12,280
	1995	.999*	.0002	.9983	.99	12,291
	1996	.999*	.0003	.9987	.99	12,291
	1997	.998	.0002	.9978	.99	12,291
	1998	.998	.0002	.9984	.99	12,291
	1999	1.00 ***	.0003	1.0011	.99	12,293
	2000	.999***	.0003	.9998	.99	12,294
	2001	1.001	.0001	1.0024	.99	12,294
	2002	1.001	.0001	1.0017	.99	12,294
	2003	1.00*	.0001	1.0009	.99	12,293
	2004	1.001	.0001	1.0027	.99	12,292
	2005	1.001	.0001	1.0031	.99	12,293
	2006	1.001	.0001	1.0027	.99	12,993
	2007	1.001	.0002	1.0039	.99	12,259
Hungary	1990	1.055	.0019	1.1215	.99	3,121
	2001	1.034	.0022	1.0806	.99	3,121
	2011	1.028	.0025	1.0674	.99	3,121
Luxembourg	1851	.941*	.0264	.9557	.93	116
	1871	.993**	.0276	1.0779	.92	116
	1880	1.005 **	.0306	1.0667	.95	116
	1890	1.048	.0183	1.0855	.93	116
	1900	1.109	.0324	1.2833	.96	116
	1910	1.079	.0187	1.1803	.99	116
	1922	1.01**	.0123	1.0473	.97	116
	1930	1.103	.019	1.2406	.98	116
	1935	1.001***	.007	1.0059	.99	116
	1947	1.014*	.0061	1.0355	.99	116
	1960	1.069	.0132	1.1725	.98	116
	1970	1.044	.0156	1.1198	.97	116
	1981	1.016**	.0144	1.0576	.98	116
	1991	.993 **	.0104	.9964	.99	116
	2001	0.955	.0076	.9199	.99	116
	2002	0.994	.0015	.989	.99	116
	2003	.995**	.0034	.9926	.99	116
	2004	.997**	.0018	.9947	.99	116
	2005	.996**	.0024	.9926	.99	116
	2006	.997**	.0015	.9961	.99	116
	2007	0.992	.0029	.9851	.99	116
	2008	0.992	.0023	.985	.99	116
	2009	.996**	.0019	.994	.99	116
	2010	.996*	.0016	.9924	.99	116
	2011	.999**	.0015	1.00	.99	11

Table 3b. Model A Estimates (Country: Malta; Different Years)

Country	Year	β	Robust s.e.	θ	R^2	N. obs.
Malta	1921	0.836*	.0770	.7891	.89	51
	1931	1.008**	.0151	1.0306	.99	54
	1948	.912**	.0582	.9684	.86	54
	1957	1.018*	.0249	1.0808	.96	55
	1967	.987**	.0163	.9853	.99	56
	1985	.936**	.0423	.9732	.90	59
	1995	.941*	.0265	.9407	.94	63
	2000	.991**	.0057	.9840	.99	67
	2001	.997**	.0015	.9959	.99	68
	2002	.995*	.0010	.9920	.99	68
	2003	.996	.0008	.9931	.99	68
	2004	.997	.0008	.9950	.99	68
	2005	.992**	.0220	1.0055	.98	68
	2006	.998**	.0013	.9967	.99	68
	2007	1.01	.0035	1.0210	.99	68
	2008	1.001**	.0012	1.0027	.99	68
	2009	1.003*	.0016	1.0069	.99	68

Table 4. Model B Estimates for the Five Countries

Country	γ	Robust s.e.	Time span	N. Obs.
Botswana	.99569**	.0149	10	460
Germany	1.00026	.0002	15	172,049
Hungary	1.03855	.0013	31	9,363
Luxembourg	1.00919 **	.0065	190	2,900
Malta	.97780**	.0113	108	1,071

Table 5a. The Zipf's and Gibrat's Parameters (Countries: Botswana, Germany, Hungary and Luxembourg; Different Years)

Country	Year	q-coefficient	Robust s.e.	β	θ	β_{BIG}	θ_{BIG}	β_{Small}	θ_{Small}	N. Obs.
Botswana	2001	1.137	.0746	-	-	-	-	-	-	464
	2011	1.173	.0746	.995**	1.0078	.979**	1.000	0.761	1.011	461
Germany	1993	1.399	.0178	-	-	-	-	-	-	12,280
	1994	1.397	.0178	.999	1.0009	.996	.994	1.001**	1.009	12,291
	1995	1.396	.0178	.999*	.9983	.996	.994	1.00**	1.004	12,291
	1996	1.394	.0177	.999*	.9987	.997	.995	1.001**	1.006	12,291
	1997	1.392	.0177	.998	.9978	.997	.994	.999**	1.002	12,291
	1998	1.390	.0177	.998	.9984	.997	.994	.997**	1.000	12,293
	1999	1.390	.0177	1.00 ***	1.0011	.998	.996	1.004	1.012	12,294
	2000	1.390	.0177	.999***	.9998	.998	.998	.998**	.998	12,294
	2001	1.391	.0177	1.001	1.0024	.999	.999	1.002	1.006	12,294
	2002	1.392	.0177	1.001	1.0017	.999**	.999	1.00**	1.002	12,294
	2003	1.392	.0177	1.00*	1.0009	.999**	1.001	.998	.998	12,293
	2004	1.393	.0177	1.001	1.0027	1.000**	1.001	1.001	1.004	12,293
	2005	1.396	.0178	1.001	1.0031	1.001	1.002	1.001**	1.003	12,293
	2006	1.398	.0178	1.001	1.0027	1.001	1.001	.999**	1.000	12,293
	2007	1.401	.0178	1.001	1.0039	1.002	1	1.001	1.005	12262
Hungary	1980	1.129	.0285	-	-	-	-	-	-	3,121
	1990	1.186	.0300	1.055	1.12	1.018	1.046	1.076	1.194	3,121
	2001	1.223	.0309	1.034	1.08	0.993	.999	1.054	1.151	3,121
	2011	1.258	.0316	1.028	1.06	1.005	1.020	1.010**	1.070	3,154
Luxembourg	1821	.5031	.0660	-	-	-	-	-	-	116
	1851	.4965	.0652	.941*	.9557	.949**	.978	.730	.764	116
	1871	.5154	.0676	.993**	1.0779	.962**	1.070	.887**	.981	116
	1880	.5350	.0702	1.005 **	1.0667	1.003**	1.060	.811**	.871	116
	1890	.5881	.0772	1.048	1.0855	.995**	1.160	.944**	.965	116
	1900	.6744	.0885	1.109	1.2833	1.115	1.340	.947**	.969	116
	1910	.7350	.0965	1.079	1.1803	1.091	1.210	.986**	1.035	116
	1922	.7500	.0984	1.01**	1.0473	0.998**	1.025	1.042**	1.140	116
	1930	.8377**	.1100	1.103	1.2406	1.090	1.240	1.01**	1.048	116
	1935	.8391**	.1101	1.001***	1.0059	.986**	.976	1.033*	1.081	116
	1947	.8543**	.1121	1.014*	1.0355	1.013**	1.034	.982**	1.009	116
	1960	.9252**	.1214	1.069	1.1725	1.011**	1.059	1.006**	1.109	116
	1970	.9735**	.1278	1.044	1.1198	0.965	.963	1.016**	1.162	116
	1980	.9923**	.1302	1.016**	1.0576	0.942	.922	1.022**	1.150	116
	1991	.9803**	.1287	.993 **	.9964	0.949	.920	1.053**	1.145	116
	2001	.9409**	.1235	0.955	.9199	0.949	.907	.910	.871	116
	2002	.9365*	.1229	0.994	.989	.998**	.997	.990**	.983	116
	2003	.9330**	.1225	.995**	.9926	1.00**	1.00	.997**	1.00	116
	2004	.9302**	.1221	.997**	.9947	1.00**	1.00	1.00**	1.020	116
	2005	.9270**	.1217	.996**	.9926	.995**	.992	.992**	.989	116
	2006	.9253**	.1215	.997**	.9961	1.001	1.00	.994**	.991	116
	2007	.9195**	.1207	0.992	.9851	0.994	.988	.974**	.955	116
	2008	.9140**	.1200	0.992	.985	.999**	.998	.976	.955	116
	2009	.9120**	.1197	.996**	.994	1.00**	1.00	.989**	.981	116
	2010	.9094**	.1194	.996*	.9924	1.00**	1.00	.986*	.974	116

	2011	.9094**	.1194	.999**	1.00	1.00**	1.00	.997**	.996	116

Table 5b. The Zipf's and Gibrat's Parameters (Country: Malta; Different Years)

Country	Year	*q*-coefficient	Robust s.e.	β	θ	β_{BIG}	θ_{BIG}	β_{Small}	θ_{Small}	N. Obs.
Malta	1901	.9883**	.1957	-	-	-	-	-	-	51
	1921	.8799**	.1693	0.836*	.7891	.893**	0.874	.487	.624	54
	1931	.8838**	.1700	1.008**	1.0306	.959**	0.965	1.067**	1.184	54
	1948	.8591**	.1638	.912**	.9684	.714	0.852	.689**	.678	55
	1957	.8760**	.1655	1.018*	1.0808	.769	0.673	1.073**	1.238	56
	1967	.8939**	.1645	.987**	.9853	.961**	0.955	.954**	.940	59
	1985	.8305**	.1479	.936**	.9732	.573	0.510	.915**	1.02	63
	1995	.8038**	.1388	.941*	.9407	.654	0.656	.923**	.912	67
	2000	.8005**	.1372	.991**	.9840	.996**	1.00	1.003**	1.009	68
	2001	.7985**	.1369	.997**	.9959	.996**	0.993	.997**	.996	68
	2002	.7956**	.1364	.995*	.9920	.994**	0.988	.993*	.986	68
	2003	.7932**	.1360	.996	.9931	.998**	.996	.993*	.987	68
	2004	.7914**	.1357	.997	.9950	.997**	.996	.995*	.990	68
	2005	.7883**	.1352	.992**	1.0055	.954**	.999	.969**	.985	68
	2006	.7881**	.1351	.998**	.9967	1.00**	1.010	.994*	.988	68
	2007	.7940**	.1361	1.01	1.0210	1.01	1.016	1.02	1.042	68
	2008	.7944**	.1362	1.001**	1.0027	.999**	1.00	1.00**	1.00	68
	2009	.7963**	.1365	1.003*	1.0069	.997**	.996	1.01	1.016	68

Table 6. Relationship between Zipf/Rank-Size and Generalized Gibrat

Country	Year	*q*	*q* trend	Gibrat	θ	Generalized Gibrat validity
Botswana	2001-2011	>1	↑	TRUE (β=1)	$\theta_{BIG}<\theta_{small}$	YES (statement b))
Germany	1993-1999	>1	↓	WEAK[28] (β=1)	$\theta_{BIG}<\theta_{small}$	YES (statement b))
	2000-2007	>1	↑	FALSE ($\beta\neq1$)	$\theta_{BIG}<\theta_{small}$	NO
Hungary	1980-2011	>1	↑	FALSE ($\beta\neq1$)	$\theta_{BIG}<\theta_{small}$	NO
Luxembourg	1821-1930	<1	↑	TRUE (β=1)	$\theta_{BIG}>\theta_{small}$	YES (statement c))
	1935-2011	=1	↔	TRUE (β=1)	$\theta_{BIG}=\theta_{small}$	YES (statement a))
Malta	1901-2009	=1	↔	TRUE (β=1)	$\theta_{BIG}=\theta_{small}$	YES (statement a))

28 Notice that weak Gibrat's law means that Gibrat's law holds only for few years.

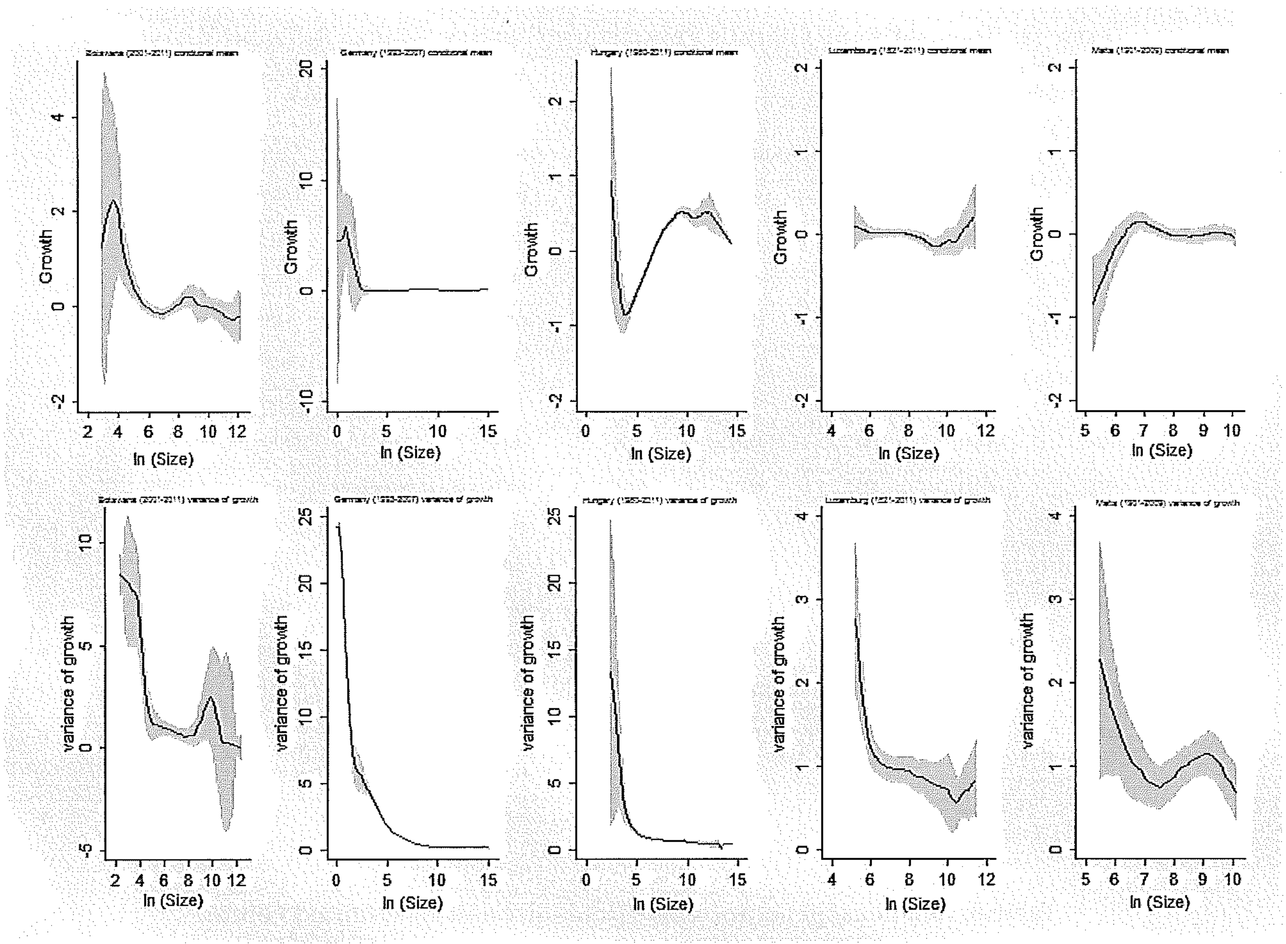

Figure 1. Non-parametric Estimator for Conditional Mean and Variance (Countries: Botswana, Germany, Hungary, Luxembourg and Malta; Different Years)

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